MARKET MILL DEPENDENCE PATTERN IN THE STOCK MARKET: MODELING OF PREDICTABILITY AND ASYMMETRY VIA MULTI-COMPONENT CONDITIONAL DISTRIBUTION

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Abstract

Recent studies have revealed a number of striking dependence patterns in high frequency stock price dynamics characterizing probabilistic interrelation between two consequent price increments x (push) and y (response) as described by the bivariate probability distribution $\mathcal{P}(x, y)$ [1, 2, 3, 4]. There are two properties, the market mill asymmetries of $\mathcal{P}(x, y)$ and predictability due to nonzero z-shaped mean conditional response, that are of special importance. Main goal of the present paper is to put together a model reproducing both the z-shaped mean conditional response and the market mill asymmetry of $\mathcal{P}(x, y)$ with respect to the axis $y = 0$. We develop a probabilistic model based on a multi-component ansatz for conditional distribution $\mathcal{P}(y|\, x)$ with push-dependent weights and means describing both properties. A relationship between the market mill asymmetry and predictability is discussed. A possible connection of the model to agent-based picture is outlined.

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1 Introduction

This paper is based on recent results on high frequency conditional dynamics in the stock market [1, 2, 3, 4]. In [2, 3, 4] we have described several new dependence structures characterizing high frequency stock dynamics - the market mill patterns corresponding to various asymmetries characterizing the bivariate probability distribution $\mathcal{P}(x, y)$ of two consecutive price increments x (push) and y (response). We have also discussed a number of effects that are best described in terms of the moments of conditional distribution $\mathcal{P}(y|\,x)$ of response y at given push x. All the conditional moments studied (mean, standard deviation, skew,normalized hypercumlant) reveal pronounced dependence on push x. A nonzero z-shaped mean conditional response implying probabilistic predictability of a price increment from knowing the previous one is of special interest. In this paper we will focus on building a probabilistic model for both nonlinear z-shaped conditional mean response and corresponding market mill asymmetry pattern of $\mathcal{P}(x, y)$ with respect to the axis $y = 0$.

The issue of price predictability is perhaps the most important one in theoretical finance. In the ideally efficient market predicting the future price increment using the historical data, e.g. the value of the preceding increment, is impossible. If one describes price dynamics as a stochastic process, it should have zero conditional mean $\langle y \rangle_x = 0$, i.e. be a martingale, see e.g. [5, 6, 7]. Let us stress that an existence of nonlinear dependence patterns related to the higher moments of conditional distribution does not contradict the martingale property and thus the weak form market efficiency. Such dependencies were intensively studied, especially in the framework of conditional regressive dynamics [7, 8].

An appearance of nonlinear mean conditional response shows that within the standard paradigm of regression models formulated as a noisy mapping of push x into response y, i.e. $y = f(x) + \varepsilon$, the mapping $f(x) = \langle y \rangle_x$ is nonlinear. Thus we are dealing with a nonlinear dynamical system, see e.g. [9, 10]. In financial applications a particular class of such models, the threshold autoregression models [11] were used in describing the properties of interest rates, see e.g. [12].

At a fundamental level it is necessary to describe the nonlinear mean conditional response in terms of market inefficiency leading to probabilistic predictability. Some examples of such predictability were discussed in [6]. Recently the issue of probabilistic predictability was discussed in the context of agent-based modeling of financial market dynamics [13].

At a phenomenological level we need to build a probabilistic model explaining the origin of both nonlinear predictability of the mean conditional response and the corresponding market mill pattern within a simple and intuitive probabilistic model. This is the main objective of the present paper.

When constructing a probabilistic model describing both the nonlinear conditional response and the market mill phenomena we shall employ a step-by-step approach. First, we provide an analytical description of the mean conditional response $\langle y \rangle_x$. Second, we consider a simple dynamical system's version of noisy conditional dynamics characterized by the observed nonlinear mean conditional response dressed by an additive noise. We show that this picture does not allow to reproduce the market mill asymmetry associated with conditional response. Finally, we present a version of noisy conditional dynamics characterized by the push-dependent mixture of conditional distributions allowing to reproduce both the nonlinear mean conditional response and the corresponding market mill asymmetry.

The outline of the paper is as follows.

In Section 2 we start with describing basic quantitative characteristics of the asymmetry of conditional response and describe an algorithm allowing to reconstruct a full bivariate push-response distribution $\mathcal{P}(x, y)$ from some model conditional distribution $\mathcal{P}(y|\, x)$. In paragraph **2.2** we describe a single-component conditional distribution corresponding to conventional noisy conditional dynamics that, by construction, reproduces an observed nonlinear dependence of mean conditional response. We show that this model gives rise to an asymmetry pattern very different from the market mill one. In paragraph **2.3** we propose a multicomponent conditional distribution of response at given push $\mathcal{P}(y|\, x)$. Its simple version with constant weights considered in paragraph **2.3.1** is shown to reproduce the market mill pattern but not the nonlinear mean conditional response. A version with push-dependent weights considered in paragraph **2.3.2** is shown to produce both the market-mill shaped asymmetry of the asymmetric component of $\mathcal{P}(x, y)$ and the nonlinearity of the conditional mean response $\langle y \rangle_x$. In paragraph **2.4** we summarize the model-dependent relationships between the nonlinear predictability and the market mill pattern. We proceed with comments on possible relation of the proposed probabilistic model to agent-based modeling of financial dynamics in paragraph **2.5** and conclude the section with a discussion of the limitations of the proposed probabilistic model in paragraph **2.6**.

In Section 3 we summarize the results of the present paper.

2 Modeling the structure of conditional response

2.1 General considerations

At the basic level a probabilistic interrelation of push x and response y is described by the bivariate distribution $\mathcal{P}(x,y)$. As already mentioned in the Introduction, in preceding papers [2, 3, 4] we have described a number of interesting effects originating in the particular features of this distribution. In the present paper we shall focus on building a model describing two such phenomena:

- Predictability of response at given push due to nontrivial nonlinear z-shaped conditional mean response $\langle y \rangle_x$.
- Market mill asymmetry of $\mathcal{P}(x,y)$ with respect to the axis $y=0$.

These effects are illustrated in Figs. 1,2 correspondingly.

Both effects originate from the nontrivial asymmetric component $\mathcal{P}^a(x,y)$ of the distribution $\mathcal{P}(x,y)$:

$$\mathcal{P}^a = \frac{1}{2}\left(\mathcal{P}(x,y) - \mathcal{P}(x,-y)\right) \tag{1}$$

The market mill dependence pattern refers to the specific shape of the positive component $\mathcal{P}^{a(p)}(x,y) \equiv \mathcal{P}^a(x,y) \cdot \Theta\left[\mathcal{P}^a(x,y)\right]$ of $\mathcal{P}^a(x,y)$, where Θ is a Heaviside step function. In turn, an emergence of nonlinear z-shaped mean conditional response $\langle y \rangle_x$ is quantified by the corresponding properties of the asymmetric component $\mathcal{P}^a(y|\,x)$ of the conditional distribution $\mathcal{P}(y|\,x) \equiv \mathcal{P}(x,y)/\mathcal{P}(x)$:

$$\langle y \rangle_x = \int dy\, y\, \mathcal{P}^a(y|\,x) \tag{2}$$

In constructing a model describing both nonlinear conditional response and market mill phenomena we shall employ a step-by-step approach.

In both above-described cases the specification of noisy conditional dynamics is made in terms of model conditional response distribution at given push $\mathcal{P}(y|\,x)$. To establish a quantitative link between the hypothesized conditional distribution $\mathcal{P}(y|\,x)$ and the full distribution $\mathcal{P}(x,y)$ one has, as follows from the definition of the conditional distribution $\mathcal{P}(y|\,x) \equiv \mathcal{P}(x,y)/\mathcal{P}(x)$, to reconstruct the marginal distribution $\mathcal{P}(x)$. This can be done by exploiting the fact that marginal distributions of push $\mathcal{P}(x)$ and response $\mathcal{P}(y)$ are identical, so that the marginal distribution satisfies, for given conditional distribution $\mathcal{P}(y|\,x)$, the following integral equation on $\mathcal{P}(x)$:

$$\mathcal{P}(y) \;=\; \int dx\, \mathcal{P}(x)\, \mathcal{P}(y|\,x) \tag{3}$$

In what follows Eq. (3) will be used for numerical reconstruction of the marginal distribution.

2.2 Single-component conditional distribution

Let us first consider a single-component model conditional distribution of the form

$$\mathcal{P}(y|\,x) \equiv \mathcal{P}(y - m(x))\,, \tag{4}$$

where $m(x) \equiv \langle y \rangle_x$ is a mean conditional response having [2] the characteristic z - shaped form. To be concrete let us parametrize the push dependence of mean conditional response in the case where x and y correspond to price increments in 3-minutes time intervals as follows:

$$m(x) \equiv \langle y \rangle_x \;=\; \left[0.14 - 0.24\,|\,x|^{\,0.15}\right] \cdot x \tag{5}$$

In Fig. 3 we plot the analytical approximation (5) together with corresponding market data from Fig. 1.

To completely specify the conditional dynamics (6,4) we have to choose a functional form of the distribution $\mathcal{P}(\varepsilon)$, where $\varepsilon \equiv y - m(x)$. Our choice is a Laplace distribution $\mathcal{P}^L(\varepsilon) = (0.5/\sigma)\exp(-|\,\varepsilon|/\sigma)$ with[3] $\sigma = \$\,0.052$. The model conditional distribution is thus given by $\mathcal{P}(y|\,x) = \mathcal{P}^L(y - m(x))$.

Using Eq. (3) one can check that it is solved by $\mathcal{P}(x) = \mathcal{P}^L(x)$, so that the full bivariate probability distribution $\mathcal{P}(x,y)$ is completely specified. Its antisymmetric component is plotted in Fig. 4 (a). We conclude that reconstructing the conditional dynamics as described by the single-component distribution (4) with appropriate push - dependent weight (5) leads to the

[3] The normalization is at the mean absolute price increment at 3-minutes scale.

two-dimensional asymmetry pattern very different from the market mill one in Fig. 2, so here one has a model with proper nonlinear mean conditional response but with a wrong asymmetry pattern.

Let us note that the probabilistic model (4,5) corresponds to a dynamical system with nonlinear mapping $x \Longrightarrow m(x)$ dressed by additive noise ε with distribution $\mathcal{P}(\varepsilon)$ and zero mean $\langle\varepsilon\rangle = 0$:

$$y = m(x) + \varepsilon \tag{6}$$

We see that a description in terms of a dynamical system (6) explains (by construction) the predictability related to the nonlinear z-shaped dependence of the conditional mean response on push but leads to the push-response asymmetry structure very different from the observed market mill shape.

2.3 Multi-component conditional distribution

Let us now construct a conditional distribution $\mathcal{P}(y|\,x)$ ensuring the z-shaped mean conditional response in a different fashion. From the analysis of [2, 3, 4] we know that the asymmetry in question is relatively weak, so that a dominant component of $\mathcal{P}(y|\,x)$ should be symmetric with respect to the reflection $y \to -y$ and, correspondingly, its dominant peak should be at $y = 0$. In addition to the symmetric component $\mathcal{P}^0(y|\,x)$ having a constant weight w^0 the distribution should include two asymmetric components $\mathcal{P}^{\pm}(y|\,x)$ having push-dependent weights $w^{\pm}(x)$:

$$\mathcal{P}(y\,|\,x) \;=\; w^+(x)\,\mathcal{P}^+(y\,|\,x) + w^0\,\mathcal{P}^0(y\,|\,x) + w^-(x)\,\mathcal{P}^-(y\,|\,x) \tag{7}$$

where $w^+(x) + w^0 + w^-(x) = 1$. Below we shall take $w^0 = 0.85$. The distributions $\mathcal{P}^{\pm}(y|\,x)$ have peaks at $y_*^{\pm}$ such that $\mathrm{sign}(y_*^{\pm}) = \pm\mathrm{sign}(x)$, so that the component $\mathcal{P}^+(y|\,x)$ corresponds to a trend-following response, whereas the component $\mathcal{P}^-(y|\,x)$ corresponds to a contrarian one. Below we shall use a simple parametrization of $m^{\pm}(x)$ ensuring these properties:

$$\begin{aligned} \langle y\rangle_x^+ \;\equiv\; m^+(x) \;&=\; (1+q)\,x \\ \langle y\rangle_x^0 \;\equiv\; m^0(x) \;&=\; 0 \\ \langle y\rangle_x^- \;\equiv\; m^-(x) \;&=\; -(1-q)\,x\,, \end{aligned} \tag{8}$$

where $\langle y\rangle_x^+ \equiv \int dy\,y\,\mathcal{P}^+(y\,|\,x)$, and q is a parameter responsible for generating the asymmetry in question, for which we will take a value of $q = 0.15$[4]. In

[4] The values of parameters contained in the model distribution $\mathcal{P}(y|\,x)$ in the expressions for weights $w^{\pm}$ and parameters of the component distributions are fixed by fitting the

Fig. 5 we plot the full conditional distirbution $\mathcal{P}(y|\, x = 0.1)$ and its three components using the weights $w^0 = 0.85$, $w^+ = 0.015$ and $w^- = 0.135$.

A simple calculation shows that the mean conditional response corresponding to Eq. (7) takes the following form:

$$\langle y \rangle_x = \left[(w^+(x) - w^-(x)) + q\, (w^+(x) + w^-(x)) \right]\, x \tag{9}$$

2.3.1 Constant weights $w^\pm$

It is instructive to start with the case of constant weights $w^\pm$. From Eq. (9) we see that in this case one gets a purely linear correlation between the conditional mean response and the push. To analyze the two-dimensional asymmetry pattern one has to specify the functional form of the distributions $\mathcal{P}^{\pm,\,0}(y|\, x)$. Let us assume that all three are Laplace distributions

$$\mathcal{P}^{\pm,\,0}(y|\, m^{\pm,\,0}(x), \sigma) \;=\; \frac{1}{2\sigma} \exp\left\{ -|y - m^{\pm,\,0}(x)|/\sigma \right\} \tag{10}$$

with common width parameter σ. We choose $w^\pm = 0.075$, $q = 0.15$ and $\sigma = \$\,0.052$. Using the consistency condition Eq. (3) we have checked that the distribution (7) with constant weights $w^\pm$ indeed solves (3) so that we can reconstruct the full two-dimensional distribution $\mathcal{P}(x, y)$ and its asymmetric component $\mathcal{P}^a$. The resulting asymmetry pattern is shown in Fig. 4 (b). We see that it has a characteristic market mill shape. We conclude that a model based on the distribution (7) with constant weights $w^\pm$ does reproduce the market mill asymmetry pattern but does not reproduce the nonlinear z-shaped mean conditional response.

2.3.2 Push-dependent weights $w^\pm(x)$

The experimentally observed dependence does have a pronounced nonlinear z-shaped form, see Fig. 1 and [2], so we have to correct our model in order to reproduce it. In fact, to generate such nonlinear dependence it is sufficient to consider the case of push-dependent weights $w^\pm(x)$. To reproduce the z-shaped pattern of Fig. 1 we have to ensure a bias towards trend-following behavior, (i.e positive slope of $\langle y \rangle_x$ vs the push) at small nonzero x and a bias towards contrarian behavior (negative slope of $\langle y \rangle_x$ vs the push) at large x.

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resulting mean conditional response to the market data corresponding to the time scale of 3 minutes.

The components of the conditional distribution Eq. (7) responsible for trend-following and contrarian behavior are obviously $\mathcal{P}^+(y|\,x)$ and $\mathcal{P}^-(y|\,x)$ correspondingly, so we have to choose some appropriate parametrization of the weights $w^\pm$. A simple illustration leading to the conditional mean response shape qualitatively similar to that in Fig. 1 is provided by

$$w^\pm(x) \;=\; \frac{1-w^0}{2}(1 \mp |\,x|) \;\;\Longrightarrow\;\; \langle y\rangle_x \;=\; (1-w^0)\,x\,(q-|\,x|) \tag{11}$$

From Eq. (11) we see that the role of the parameter q is in fixing the scale at which the nonlinear mean conditional response $\langle y\rangle_x$ changes its sign.

To reproduce the shape of conditional mean response close to the observed one, see Fig. 1, we shall use a somewhat more complex parametrization of the weights $w^\pm(x)$:

$$\begin{aligned} w^-(x) &= \min\left(w^a + \left(1-w^0-w^a\right)\left(\frac{|\,x|}{0.3}\right)^p,\; 1-w^0\right) \\ w^+(x) &= 1-w^0-w^-(x) \end{aligned} \tag{12}$$

Choosing $w^0 = 0.85$, $w^a = 0.05$, $q = 0.25$, $p = 0.5$ and $\sigma = \$\,0.052$ leads to the above-introduced parametrization of conditional response in Eq. (5). The resulting asymmetry pattern is plotted in Fig. 4 (c) and the push dependence of the weights $w^\pm(x)$ is illustrated in Fig. 6. Let us note that such fine details of the structure of the asymmetry of the empirical distribution $\mathcal{P}(x, y)$ shown in Fig. 2 as the shape of equiprobability lines and the varying form of mill blades are, as seen in Fig. 4 (), reproduced by our model . Let us stress that a bias towards trend-following behavior at small pushes and towards contrarian behavior at large ones results from combination of x - dependent weights and means.

Let us also emphasize that although it is formally possible to rewrite the conditional dynamics described by the three-modal distribution with push-dependent weights Eq. (7) in the form of Eq. (6) with some very intricate noise distribution $\mathcal{P}(\varepsilon)$, this procedure looks extremely unnatural. In this sense Eq. (7) presents a really different view on conditional dynamics than the conventional Eq. (6). The resulting asymmetry pattern is the same as in the case of constant weights plotted in Fig. 6, so that the sought for market mill structure is indeed reproduced.

Thus a model based on the conditional distribution (7) with specially chosen push-dependent weights $w^\pm(x)$ allows to reproduce both nonlinear the mean conditional response and the market mill asymmetry pattern.

The above-discussed procedure of reconstructing the bivariate distribution $\mathcal{P}(x,y)$ is by no means unique. Let us illustrate this by presenting another model also based on the three-component ansatz for the conditional distribution $\mathcal{P}(y|\,x)$. In this model the weights $w^{\pm}$ are the same as in Eq. (12) and

$$
\begin{aligned}
m^{+}(x) &= x, \qquad \sigma^{+}(x) = 0.005 + 0.065\left(\frac{\sqrt{|\,x|}}{0.3}\right)^{0}.5 \\
m^{0}(x) &= 0, \qquad \sigma^{0} = 0.03 \\
m^{-}(x) &= -x, \qquad \sigma^{-}(x) = 0.01 + 0.025\left(\frac{\sqrt{|\,x|}}{0.3}\right)^{0}.5
\end{aligned}
\tag{13}
$$

The resulting asymmetry pattern is shown in Fig. 4 (d).

Let us note that in Eqs. (12,13) the asymmetric dependence of the parameters of the component distributions is "shifted" from the means $m^{\pm}(x)$ to the widths $\sigma^{\pm}(x)$, cf. Eq. (8). All the results obtained using the model of Eqs. (12,13) are similar to those obtained with Eqs. (8,12).

The models of Eqs. (8,12) and (12,13) are just two examples from a long list of models that describe the market mill asymmetry with respect to the axis $y = 0$ and the nonlinear mean conditional response. Our choice was motivated by their transparent logical structure.

2.4 Relationship between nonlinear predictability and market mill asymmetry

At this point it is appropriate to summarize the relationship between the the phenomena of nonlinear predictability due to nontrivial push-dependent mean conditional response and the market mill asymmetry pattern characterizing the asymmetry of the bviariate distribution $\mathcal{P}(x,y)$ with respect to the axis $y = 0$. Our considerations in the paragraphs 2.2 and 2.3 have shown that this relationship is model - dependent. In particular:

- The single-component model of conditional dynamics of (4,5) describes nonlinear predictability (i.e. z-shaped mean conditional response) but gives a wrong asymmetry pattern of $\mathcal{P}(x,y)$. Thus the nonlinear mean conditional response does not constitute a sufficient condition for the existence of the market mill asymmetry.

- The multi-component model of (7,8) with constant weights $w^{\pm}$ describes the market mill asymmetry pattern but not the nonlinear dependence of the mean conditional response on push. Thus the market mill asymmetry does not constitute a sufficient condition for the existence of the nonlinear mean conditional response.

2.5 Market mill from the agent-based perspective

Let us now discuss the possible origins of the market mill asymmetry in the framework of agent-based description of financial market dynamics.

A direct link between the agent's strategies and price evolution is provided by the relation between the sign of market orders and the resulting change in price, see e.g. [14]. In the discrete time formulation this corresponds to a dependence of price increment $\delta p_t = p_{t+1} - p_t$ on cumulative sum of signed orders $\Omega_t = V_t^+ - V_t^-$ placed at time t, where $V^{\pm}$ is a volume of buy (+) and sell (-) orders. With the simplest assumption of linear impact

$$\delta p_t = \frac{1}{\lambda} \Omega_t \, . \tag{14}$$

Assuming constant proportionality coefficient λ, the probability distribution of price increments is a rescaled version of the probability distribution of the signed volume:

$$\mathcal{P}(\Omega_t) \quad \longrightarrow \quad \mathcal{P}(\delta p_t) \tag{15}$$

The distribution $\mathcal{P}(\Omega_t)$ provides a probabilistic description of agent's strategies realized through buying (selling) a certain number of stocks or just doing nothing at time t. Let us consider a simple case when such trading decisions depend on the preceding price increment δp_{t-1}, so that

$$\mathcal{P}\left(\delta p_t\right) = \frac{1}{\lambda} \mathcal{P}\left(\Omega_t | \, \delta p_{t-1}\right) . \tag{16}$$

Within this framework it is natural to classify agents into three groups, trend-following contrarian and noise, characterized by probability distributions $\mathcal{P}^+\left(\Omega_t | \, \delta p_{t-1}\right)$, $\mathcal{P}^-\left(\Omega_t | \, \delta p_{t-1}\right)$ and $\mathcal{P}^0\left(\Omega_t | \, \delta p_{t-1}\right)$ correspondingly. The distributions $\mathcal{P}^{\pm,0}\left(\Omega_t | \, \delta p_{t-1}\right)$ are biased in such a way that

$$\mathrm{sign}\left(\mathrm{E}[\, \mathcal{P}^{\pm}\left(\Omega_t | \, \delta p_{t-1}\right)]\right) = \pm \mathrm{sign}\left(\delta p_{t-1}\right)$$

and $\mathrm{E}\left[\mathcal{P}^{\pm}\left(\Omega_t | \, \delta p_{t-1}\right)\right] = 0$. The trend-followers are betting that the sign of the next price increment is on average the same as that of the previous one, the contrarians bet on sign reversal and noise traders make random decisions. Generically the yields $w^{\pm,0}$ of the trend-following, contrarian and noise strategies depend both on the sign and magnitude of δp_{t-1}. Thus

$$\mathcal{P}\left(\Omega_t \middle| \, \delta p_{t-1}\right) = \sum_{i=+,-,0} \mathcal{P}^i\left(\Omega_t \middle| \, \delta p_{t-1}\right) \tag{17}$$

which is, due to (14), precisely the conditional distribution (7). Therefore the three components of the conditional distribution $\mathcal{P}\left(\Omega_t \middle| \, \delta p_{t-1}\right)$ in Eq. (17) (trend-following, contrarian and noise) precisely correspond to the three components of the conditional distribution $\mathcal{P}(y| \, x)$ in Eq. (7).

2.6 Limitations of the model

In the present paper we have focused on building a probabilistic model describing the market mill asymmetry pattern corresponding to one particular asymmetry of $\mathcal{P}(x, y)$, that of reflection $y \to -y$. As shown in [2], the full empirical bivariate distribution is in fact characterized by several asymmetry patterns having the market mill shape, e.g. that corresponding to the reflection with respect to the axis $y = x$. We have checked that with the ansatz Eq. (7) one can not reproduce the market mill pattern corresponding to this last asymmetry. In fact, our ansatz is tailored to describe only one particular asymmetry, that of conditional response. Constructing a probabilistic model describing the full asymmetry structure of $\mathcal{P}(x, y)$ remains a task for the future.

Let us also note that even with the asymmetry pattern under consideration the ansatz Eq. (7) does not allow to reproduce all details of the empirically observed pattern. For example, if we introduce the dependence of conditional standard deviation on the push, see [3], the market mill pattern gets heavily distorted.

3 Conclusions

Let us summarize the main results obtained in the paper:

- A probabilistic model based on the multi-component model for conditional distribution $\mathcal{P}(y|, x)$ reproducing the nonlinear z-shaped mean conditional response and the market mill conditional response asymmetry pattern was constructed.

- We demonstrated that a single-component model corresponding to conventional noisy conditional dynamics with built-in z-shaped mean conditional response does not allow to reproduce the market mill conditional response asymmetry pattern, so that an existence of the z-shaped mean conditional response does not imply that of the market mill asymmetry pattern.
- Consideration of the case of push-independent weights in the multi-component model conditional distribution $\mathcal{P}(y|\, x)$ showed that the market mill asymmetry pattern can coexist with the usual linear dependence of the mean conditional response on push, so that an existence of the market mill asymmetry pattern does not imply that of the z-shaped mean conditional response.
- A possible link of the discussed probabilistic model with agent-based description of market dynamics was outlined.

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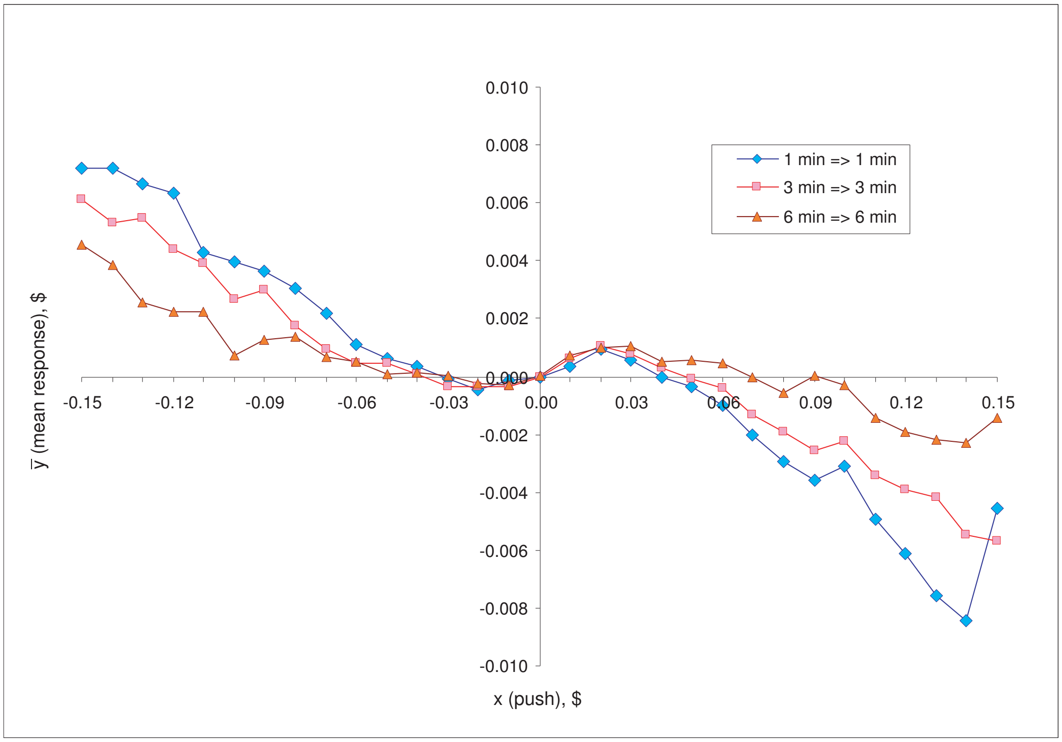

Figure 1: Mean conditional response versus push [2]

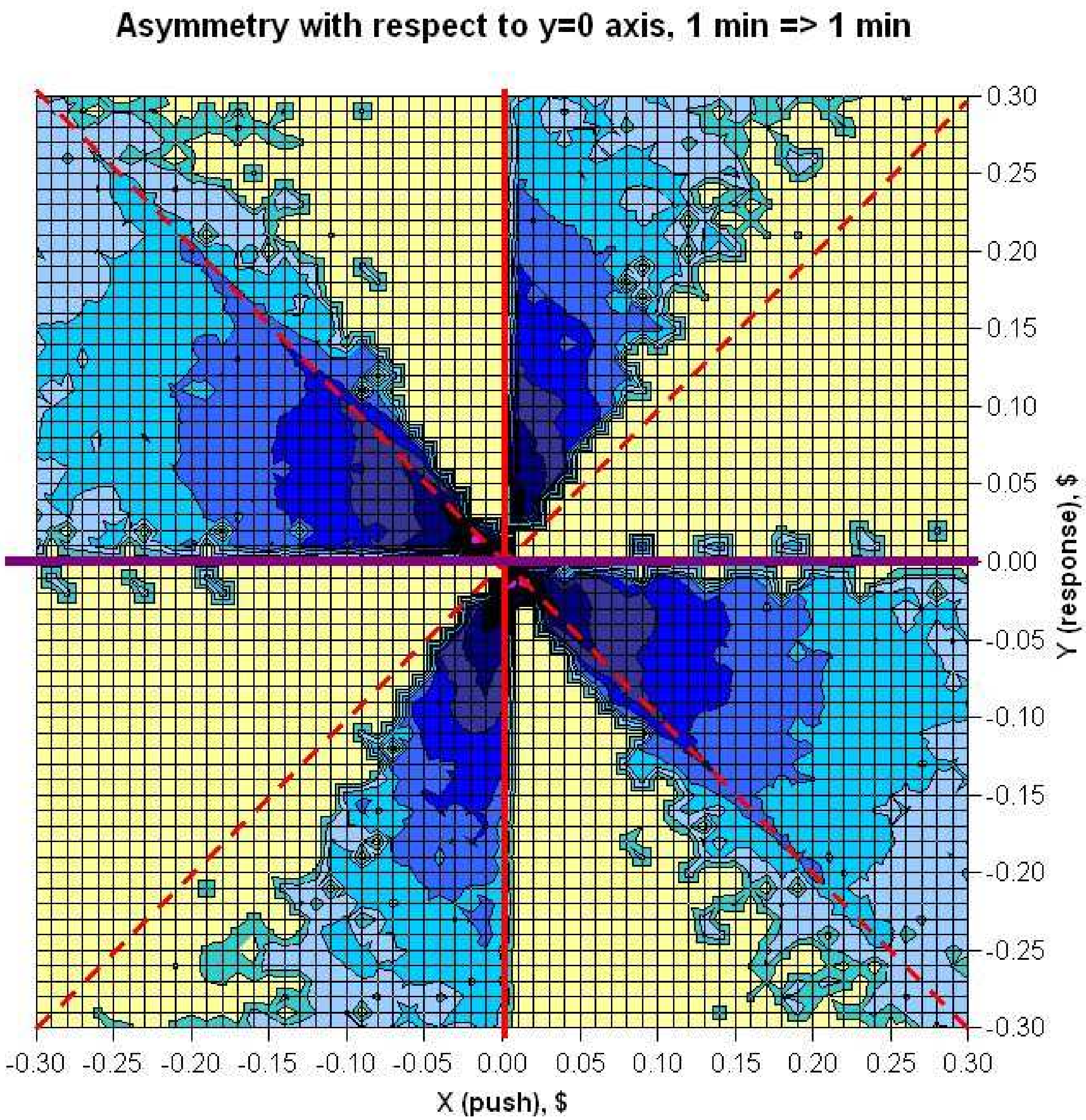

Figure 2: Two-dimensional projection of the asymmetry of the bivariate distribution $\log_2(\mathcal{P}(x,y))$ [2].

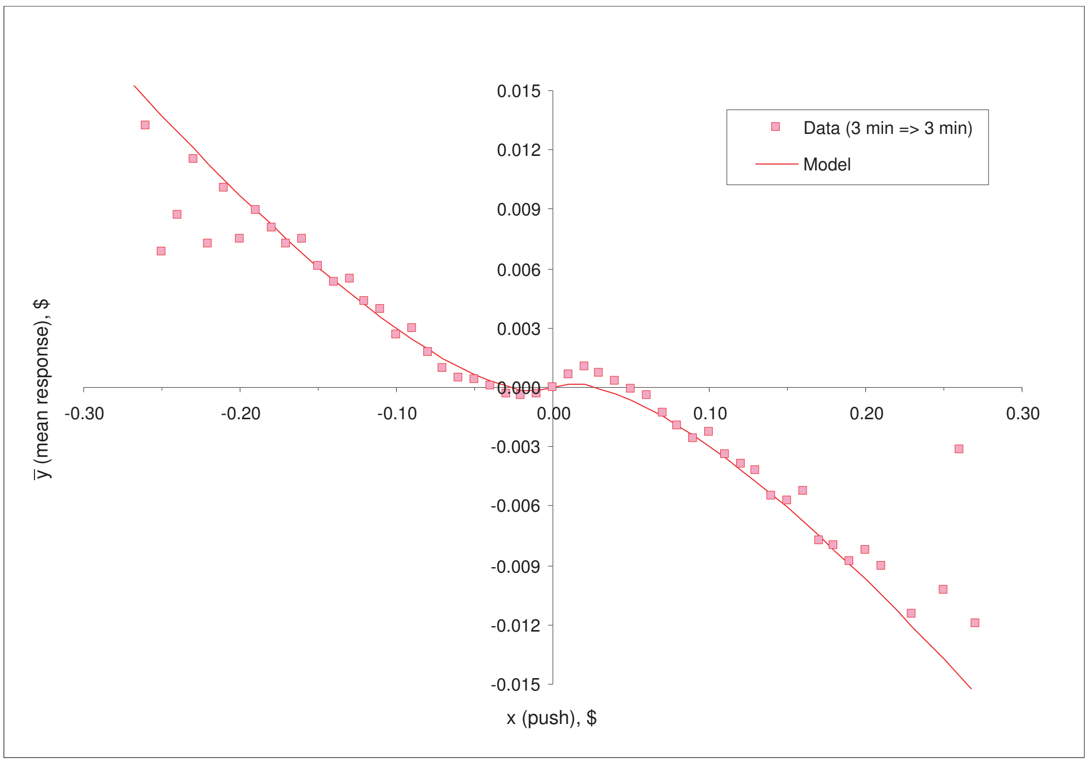

Figure 3: Mean conditional response: model versus data.

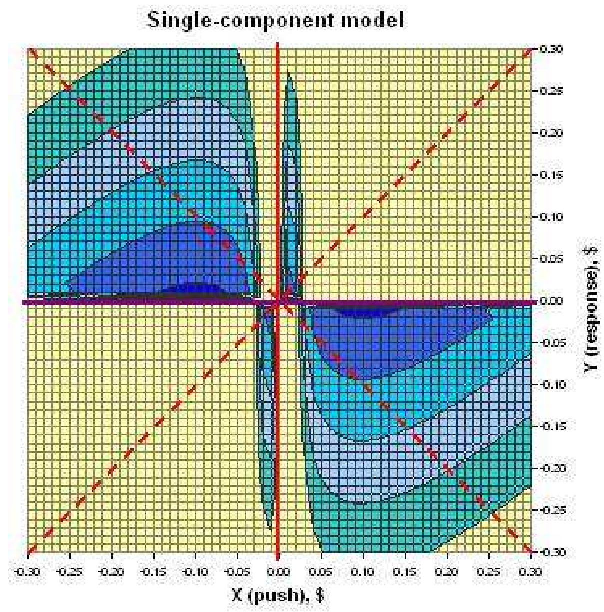

Figure 4 (a)

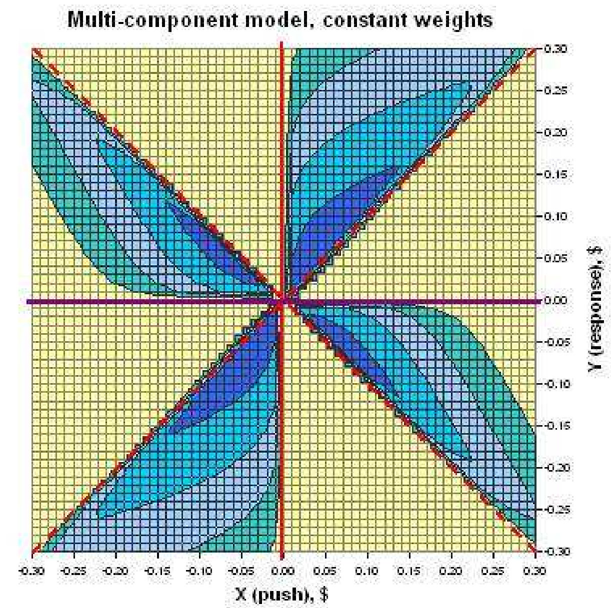

Figure 4 (b)

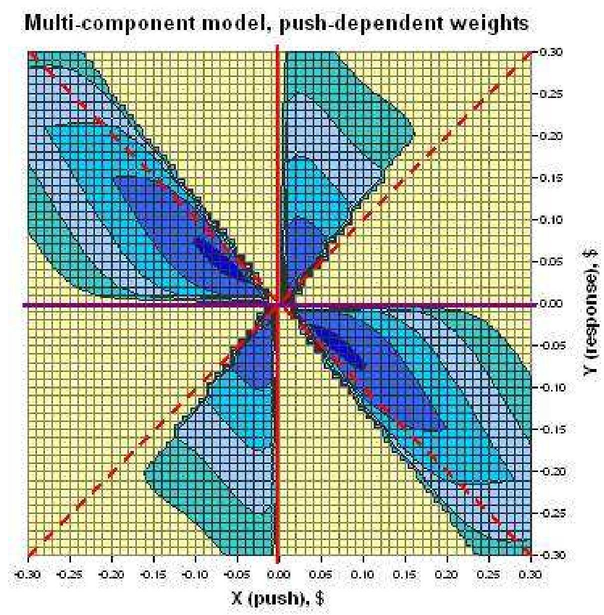

Figure 4 (c)

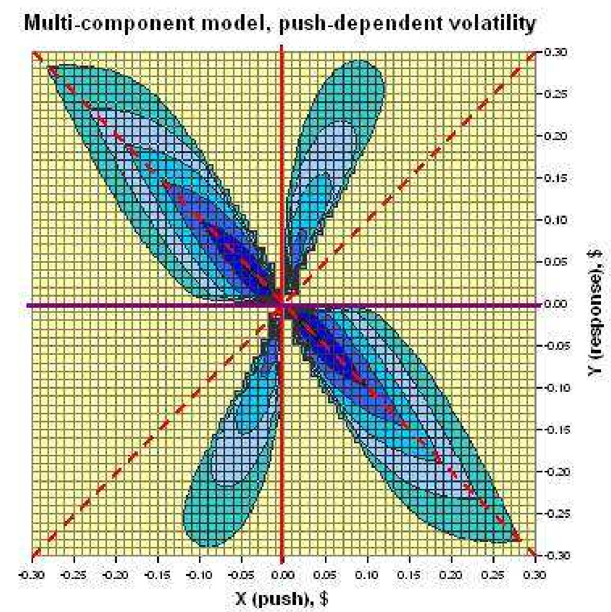

Figure 4 (d)

Figure 4: Two-dimensional projection of the asymmetry of the bivariate distribution P(x,y). (a) Single-component model. (b) Multi-component model, constant weights. (c) Multi-component model, push-dependent weights. (d) Multi-component model, push-dependent volatility.

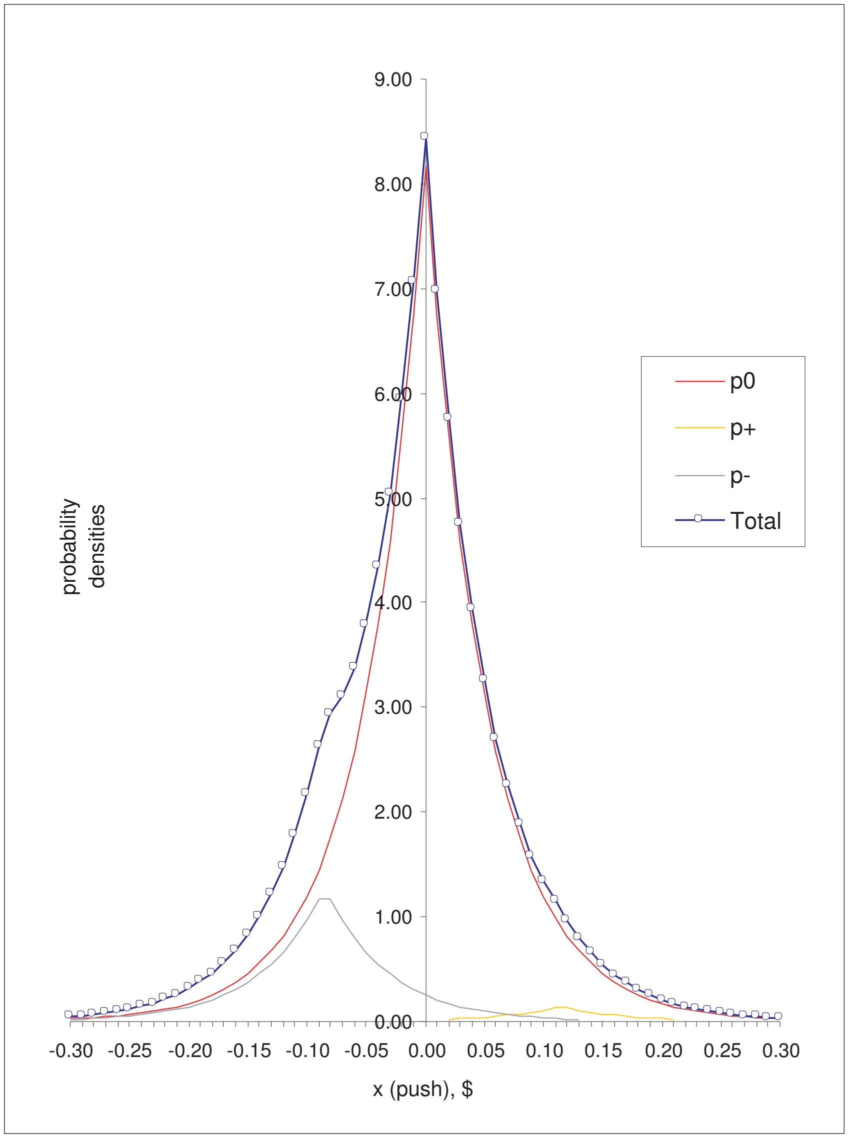

Figure 5: Components of conditional distribution $\mathcal{P}(y|\,x)$ at $x = 0.1$.

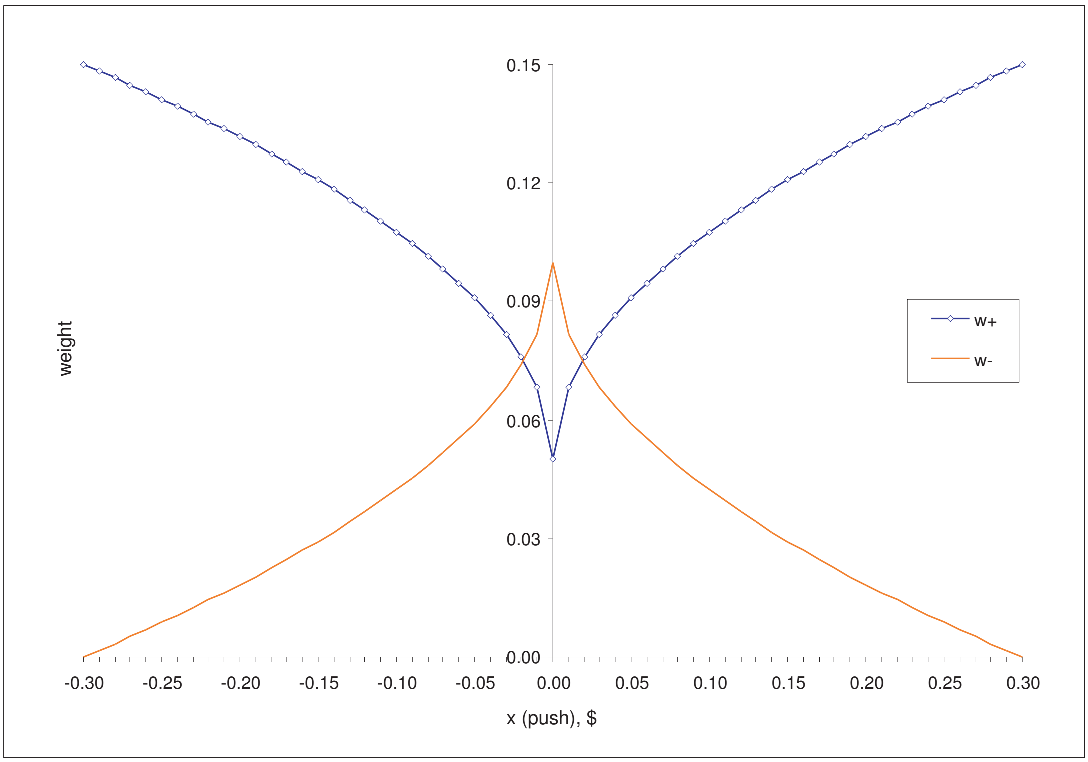

Figure 6: Weights $w^{\pm}$ of asymmetric components $\mathcal{P}^{\pm}(y|\,x)$.